Calculation of gluon and four-quark condensates from the operator expansion

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The magnitudes of gluon and four-quark condensates are found from the analysis of vector mesons consisting of light quarks (the families of ρ and ω mesons) in the 3 loops approximation. The QCD model with infinite number of vector mesons is used to describe the function $R(s)$. This model describes well the experimental function $R(s)$. Polarization operators calculated with this model coincide with the Wilson operator expansion at large Q^2. The improved perturbative theory, such that the polarization operators have correct analytical properties, is used. The result is $\langle 0|(\alpha_s/\pi)G^2|0\rangle = 0.062 \pm 0.019 GeV^4$. The electronic widths of $\rho(1450)$ and $\omega(1420)$ are calculated.

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I. INTRODUCTION

The purpose of this work is to propose a new method of calculation of gluon and other condensates in some loop approximation from analysis of families of vector mesons consisting of light quarks (ρ, ω-families).Calculation of gluon and other condensates is based on the Wilson operator expansion (OPE) (formulae (7-10)) for polarization operator $(\Pi^{(\rho)}(Q^2))_{theor}$. On the other hand, the dispersion relation for $((\Pi^{(\rho)}(Q^2))_{exp}$ (Eq.3) makes it possible to express $(\Pi^{(\rho)}(Q^2))_{Exp}$ through the measurable function $R^{(\rho)}(s)$. At large Q^2 $(\Pi^{(\rho)}(Q^2))_{Exp}$ must coincide with $(\Pi^{(\rho)}(Q^2))_{theor}$. In order to present$(\Pi^{(\rho)}(Q^2))_{Exp}$ in the form (7) we use the QCD model with an infinite number of vector mesons (MINVM) suggested and used in papers [1-3]. The MINVM was used in papers [4,5] for calculation of hadronic contribution to muon $(g-2)$-factor and $\alpha(M_Z^2)$. The accuracy of these calculations by MINVM is about 1%. It is evident that the MINVM can be used only under the integral.

This work is the continuation of Ref.[6] devoted to evaluating of the QCD parameters. In Ref.[6] the analiticity of QCD polarization operators were combined with the renormalization group and was used for investigation of hadronic τ-decay.

The calculation according to renormalization group leads to appearance of non-physical singularities. So, the one-loop calculation gives a nonphysical pole, while in the calculation in a larger number of loops the pole disappears, but a nonphysical cut appears $[-\Lambda_3^2, 0]$.[1] In Ref.[6] it was shown that there are only two values of Λ_3, such that theoretical predictions of QCD for $R_{\tau,V+A}$ (formulae (23) and (24) of [6]) agree with the experiments [7-9]. These values calculating in the three loops are the following: one conventional value $\Lambda_3^{conv} = (618 \pm 29)MeV$ and the other value of Λ_3 is $\Lambda_3^{new} = (1666 \pm 7)MeV$. The predictions of QCD consistent with the experiments [7-9] are just within these values of Λ_3. If one simply takes off the nonphysical cut and leaves the conventional value Λ_3^{conv} then discrepancy between the theory and experiment will arise. As was shown in [6], if instead of conventional value Λ_3^{conv} one chooses the value $\Lambda_3^{new} = (1565 \pm 193)MeV$, then only the physical cut contribution is sufficient to explain the experiment on hadronic τ-decay.

The new sum rules following only from analytical properties of the polarization operator were obtained in [6].These sum rules imply that there is an essential discrepancy between perturbation theory in QCD and the experiment in hadronic τ-decay at conventional value of Λ_3. If $\Lambda_3 = \Lambda_3^{new}$, this discrepancy is absent. Because Λ_3^{conv} is conventional, we will calculate the condensates for both admissible values of Λ_3; $\Lambda_3^{new} = (1565 \pm 193)MeV$ (in three loops) without the nonphysical cut corresponding $\alpha_s(-m_r^2) = 0.379 \pm 0.013$ and $\Lambda_3^{conv} = (618 \pm 29)MeV$ with the nonphysical cut corresponding $\alpha_s(-m_r^2) = 0.354 \pm 0.010$.

The paper is organized as follows. In Sec.II the QCD model with an infinite number is expanded. The polarization operator defined from experiment with the help of formulae (3-6) has the form of OPE and owing to this, it is needless to use the Borel transformation. To find the condensates it is sufficient to equate the coefficients at $1/Q^n$. In Sec.III the magnitudes of gluon and four-quark condensates are calculated from analysis of ρ-meson family without taking into account $\rho - \omega$ inter-

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[1]This is the definition of Λ_3.

ference. It is obtained $\langle 0|(\alpha_s/\pi)G^2|0\rangle = (0.074 \pm 0.023)GeV^4$ $(\Lambda_3 = 1.565GeV)$ and $\langle 0|(\alpha_s)/\pi G^2|0\rangle = (0.112 \pm 0.021)GeV^4$ $(\Lambda_3 = 0.618GeV)$.

In Sec.IV the magnitudes of gluon and four-quark condensates are calculated from analysis of ω-meson family without taking into account $\rho - \omega$ interference. It is obtained $\langle 0|(\alpha_s/\pi)G^2|0\rangle = (-0.076 \pm 0.033)GeV^4$ $(\Lambda_3 = 1.565GeV)$ and $\langle 0|(\alpha_s/\pi)G^2|0\rangle = (-0.043 \pm 0.031)GeV^4$ $(\Lambda_3 = 0.618GeV^4)$.

The magnitude of gluon condensate obtained from analysis of ω-family must be equal to the magnitude of gluon condensate from analysis of ρ-family.

This discrepancy is due to that all formulae obtained from OPE are valid only for states with pure isospin. But, owing to the vicinity of the mass of the ρ and ω-mesons the ρ-meson has a small admixture of the state with isospin $I = 0$ and the ω-meson has a small admixture of the state isospin $I = 1$. The contradiction resolves after the separation from the ρ and ω mesons of the pure states ρ_0 with the isospin $I = 1$ and ω_0 with isospin $I = 0$. It must be emphasized that strong cancellations occur in the formulae determining the condensates. For these reasons the account of the fine effects as the $\rho - \omega$ interference is essential. In Sec.V the analysis of ω-family with account of the $\rho - \omega$ interference is given. The results of the calculations in the $(0-3)$loop approximation of gluon and four-quarks condensates are presented in Table I $(\Lambda_3 = 1.565GeV)$ and in Table II $(\Lambda_3 = 0.618GeV)$. The same analysis of ρ-family is given in Sec.VI (Table III,IV). Because the values of the condensates obtained from the analysis of the ρ and ω families agree closely in Sec.IX the averaged values of the condensates (Table V,VI) are presented. From these tables it is evident that the expansion of the gluon and 4-quarks condensates in terms of α_s is very good. As a by product in Sec's.VII and VIII, we calculated the electronic widths of $\rho(1450)$ and $\omega(1420)$.

II. THE QCD MODEL WITH AN INFINITE NUMBER
OF VECTOR MESONS.

In this Section we present the QCD model with infinite number of vector mesons suggested in Refs.[1-3]. This model is a basis for the following calculations.

Let us consider at first the family of ρ-mesons. The polarization operator $\Pi^{(\rho)}$ corresponding to the ρ-meson family has the form

$$i \int d^4x e^{iqx} \langle 0|T\{j_\mu^{I=1}(x), j_\nu^{I=1}(0)\}|0\rangle = (q_\mu q_\nu - q^2 g_{\mu\nu})\Pi^{(\rho)}(Q^2), \tag{1}$$

where $Q^2 = -q^2$ and

$$j_\mu^{I=1}(x) = (1/2)[\bar{u}(x)\gamma_\mu u(x) - \bar{d}(x)\gamma_\mu d(x)] \tag{2}$$

is the current of vector mesons with isospin $I = 1$.

The dispersion relation is given by

$$\Pi^{(\rho)}(Q^2) = \frac{1}{12\pi^2} \int\limits_{4m_\pi^2}^{\infty} \frac{R^{I=1}(s)}{s + Q^2} ds \tag{3}$$

For the sake of simplicity it is written without subtractions. It is shown below that the divergent terms in (3) cancel. In the QCD model with an infinite number of narrow

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resonances with the masses M_k and and the electronic widths Γ_k^{ee}, function $R^{I=1}(s)$ has the form

$$R^{I=1}(s) = \frac{9\pi}{\alpha^2} \sum_{k=0}^{\infty} \Gamma_k^{ee} M_k \delta(s - M_k^2) \tag{4}$$

where $\alpha = 1/137$. Formula (4) obviously contradicts to experiments. Let us replace $\delta(s - M_k^2) \to (1/\pi) M_k \Gamma_k / [(s - M_k^2)^2 + M_k^2 \Gamma_k^2]$, Γ_k is the total width of k-th resonance. Then we get, instead of Eq.(4)

$$R^{I=1}(s) = \frac{9}{\alpha^2} \sum_{k=0}^{\infty} \frac{\Gamma_k^2 M_k^2}{(s - M_k^2)^2 + M_k^2 \Gamma_k^2} \tag{4a}$$

If the total widths of all resonances Γ_k are much smaller than their masses M_k, the results of the integration of (4) and (4a) with smooth functions coincide. If $M_k \Gamma_k \gg M_k^2 - M_{k-1}^2$ for $k > 3$ function (4a) is described by a smooth curve for $s > M_3^2$. When fulfilling these conditions formula (4a) is consistent with experimental data of $R^{I=1}(s)$. Formula (4) will be used only under the integral with a smooth function.

Using (4), recast (3) into the form

$$\Pi^{(\rho)}(Q^2) = \frac{3}{4\pi\alpha^2} \sum_{k=0}^{\infty} \frac{\Gamma_k^{ee} M_k}{s_k + Q^2} \tag{5}$$

The polarization operator (5) can be rearranged into a form with the separated unit operator. The remainder of the polarization operator can be associated with gluon condensate and with contribution of higher dimensional operators. To do this we transform the sum in (5) into an integral by means of the Euler-Maclaurin formula [10] beginning from $k = 1$. We have

$$\Pi^{(\rho)}(Q^2) = \frac{3}{4\pi\alpha^2} \left\{ \int\limits_{(m_u+m_d)^2}^{\infty} \frac{\Gamma_k^{ee} M_k}{s_k + Q^2} \frac{dk}{ds_k} ds_k - \int\limits_{(m_u+m_d)^2}^{s_1} \frac{\Gamma_k^{ee} M_k}{s_k + Q^2} \frac{dk}{ds_k} ds_k + \frac{\Gamma_0^{ee} M_0}{s_0 + Q^2} + \right.$$

$$\left. \frac{1}{2} \frac{\Gamma_1^{ee} M_1}{s_1 + Q^2} - \frac{1}{12} \frac{d}{dk} \left(\frac{\Gamma_k^{ee} M_k}{s_k + Q^2} \right) \bigg|_{k=1} + \frac{1}{720} \frac{d^3}{dk^3} \left(\frac{\Gamma_k^{ee} M_k}{s_k + Q^2} \right) \bigg|_{k=1} - ... \right\} \tag{6}$$

The operator expansion for $\Pi^{(\rho)}$, which is valid at high Q^2 has the form [11]

$$\Pi^{(\rho)}(Q^2) = \int\limits_{(m_u+m_d)^2}^{\infty} \frac{R_{PT}^{I=1}(s) ds}{s + Q^2} + C_2/Q^2 + C_4/Q^4 + C_6/Q^6 + ... \tag{7}$$

$$C_2 = 0 \tag{8}$$

$$C_4 = \frac{1}{24} \langle 0| \frac{\alpha_s}{\pi} G^2 |0\rangle + \frac{1}{2} (m_u \langle 0|\bar{u}u|0\rangle + m_d \langle 0|\bar{d}d|0\rangle) \tag{9}$$

$$C_6 = -\frac{1}{2} \pi \alpha_s \langle 0| (\bar{u}\gamma_\mu\gamma_5 t^a u - \bar{d}\gamma_\mu\gamma_5 t^a d)^2 |0\rangle$$

$$- \frac{1}{9} \pi \alpha_s \langle 0| (\bar{u}\gamma_\mu t^a u + \bar{d}\gamma_\mu t^a d) \sum_{q=u,d,s} \bar{q}\gamma_\mu t^a q |0\rangle. \tag{10}$$

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In the region where the operator expansion is valid, $\Pi^{(\rho)}(Q^2)$ should not differ significantly from $\Pi^{(\rho)}_{theor}(Q^2)$ (see eq.(7)). For this reason we equate the first term on the right-hand part of (6) to the first term on the right-hand part of (7).

$$\frac{3}{4\pi\alpha^2}\int\limits_{(m_u+m_d)^2}^{\infty}\frac{\Gamma^{ee}_k M_k}{s_k+Q^2}\frac{dk}{ds_k}ds_k = \frac{1}{12\pi^2}\int\limits_{(m_u+m_d)^2}^{\infty}\frac{R^{I=1}_{PT}(s)ds}{s+Q^2}\tag{11}$$

We consider the equality (11) as an ansatz that makes it possible to separate large terms associated with the unit operator from small terms associated with condensates and consider it as an equation for Γ^{ee}_k. Eq.(11) has one and only one solution[2]

$$\Gamma^{ee}_k = \frac{2\alpha^2}{9\pi}R^{(I=1)}_{PT}(s_k)M^{(1)}_k\tag{12}$$

This result follows from uniqueness of the theorem for analytic functions. The solution (12) is obtained by equating the jumps on the cut in (11). In eq.(12) and in the following formulae we use the notation

$$s_k = M^2_k,\quad M^{(l)}_k \equiv d^l M_k/dk^l,\quad s^{(l)}_k \equiv d^l s_k/dk^l\tag{13}$$

It can be proved that the function $s_k \equiv s(k)$ specified at the points $k = 0, 1, 2, ...$ can be extended to an analytic function of the complex variable k with a cut along the negative axis [3]. We will not employ the analytic properties of the function $s(k)$. It is assumed that the function $s(k)$ is continuous, differentiable with respect to k at $k = 1$. The derivatives $s^{(l)}_1$ will be considered as parameters.

The QCD model with an infinite number of vector mesons was used in [5,12] to calculate the contribution of strong interaction to anomalous magnetic moment of muon. The result was found to be

$$a^{hadr}_\mu = ((g-2)/2)_{hadr} = \frac{\alpha}{3\pi^2}\int\limits_{4m^2_\pi}^{\infty}dsK(s)R(s)/s = 678(7)\cdot 10^{-10}\tag{14}$$

The result given by eq.(14) should be compared with the recent, more precise, a^{hadr}_μ value calculated by integrating formula (14) with the cross sections measured from annihilation e^+e^- to hadrons [13-15]

$$a^{hadr}_\mu = 6847(70)\cdot 10^{-11}\quad [13]$$

$$a^{hadr}_\mu = 6831(61)\cdot 10^{-11}\quad [14]$$

$$a^{hadr}_\mu = 6909(64)\cdot 10^{-11}\quad [15]\tag{15}$$

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[2]The analogous formula in nonrelativistic quantum mechanics

$$\mid\psi_k(0)\mid^2 = \frac{m^{3/2}}{\sqrt{2}\pi^2}E^{1/2}_k\frac{dE_k}{dk}\tag{12a}$$

has accuracy $\lesssim 2\%$ for usual potential.

In addition, the QCD model with an infinite number of vector mesons was used in [4,5,12] to calculate the contribution of strong interaction to quantity $\alpha(M_z^2)$. It was found that

$$\delta\alpha_{hadr} = \frac{\alpha M_z^2}{3\pi} P \int\limits_{4m_\pi^2}^{\infty} \frac{R(s)ds}{(M_z^2 - s)s} = 0.02786(6) \qquad (16)$$

This result should be compared with the results $\delta\alpha_{hadr} = 0.02744(36)$ [16], $0.02803(65)$ [17], $0.0280(7)$ [18], $0.02754(46)$ [19], $0.02737(39)$ [20], $0.02784(22)$ [21], $0.02778(16)$ [22], $0.02779(20)$ [23], $0.02770(15)$ [24], 0.02787 (32) [25], $0.02778(24)$ [26], 0.02741 (19) [27], $0.02763(36)$ [28], and $0.02747(12)$ [29] which were obtained by calculating the integral (16) with the experimental cross section from e^+e^- into hadrons. We emphasize that the quantity $\delta\alpha_{hadr}$ is calculated in (16) with the highest accuracy.

It is important to note that the integrals which describe hadronic contributions to the $(g-2)$ factor for muon and to $\alpha(M_z^2)$ are determined by different regions: the integrals for hadronic contributions to the $(g-2)$ factor is governed by the region of small $s \sim m_\rho^2$, while the integral for hadronic contribution to $\alpha(M_z^2)$ is dominated by large $s \sim M_z^2$. From the above that accuracy of calculations by MINVM is about 1%. This accuracy is sufficient for calculations of gluon and four-quark condensates. It is obviously that MINVM can be used only under the integral.

III. MAGNITUDE OF GLUON AND FOUR-QUARK CONDENSATES FROM ANALYSIS OF ρ-MESON FAMILY

At present, three mesons of ρ-family with the masses $M_0 = 0.7711 \pm 0.0009 GeV$, $M_1 = 1.465 \pm 0.025 GeV$, $M_2 = 1.7 \pm 0.02 GeV$ have been found. The electronic width of $\rho(770)$ is $\Gamma_0^{ee} = (6.85 \pm 0.11) keV$ [30].

Comparing (6) and (7) at high Q^2 and using formula (12), we arrive at

$$C_2 = \frac{1}{12\pi^2} \left\{ - \int\limits_{(m_u+m_d)^2}^{s_1} R_{PT}^{I=1}(s)ds + R_{PT}^{I=1}(s_0)s_0^{(1)} + \right.$$

$$\left. \frac{1}{2}R_{PT}^{I=1}(s_1)s_1^{(1)} - \frac{1}{12}[R_{PT}^{I=1}(s_1)s_1^{(1)}]^{(1)} \right\} \qquad (17)$$

$$C_4 = \frac{1}{12\pi^2} \left\{ \int\limits_{(m_u+m_d)^2}^{s_1} R_{PT}^{I=1}(s)sds - R_{PT}^{I=1}(s_0)s_0 s_0^{(1)} - \frac{1}{2}R_{PT}^{I=1}(s_1)s_1 s_1^{(1)} + \right.$$

$$\frac{1}{12}[R_{PT}^{I=1}(s_1)s_1 s_1^{(1)}]^{(1)} \right\} \qquad (18)$$

$$C_6 = \frac{1}{12\pi^2} \left\{ - \int\limits_{(m_u+m_d)^2}^{s_1} R_{PT}^{I=1}(s)s^2 ds + R_{PT}^{I=1}(s_0)s_0^2 s_0^{(1)} + \frac{1}{2}R_{PT}^{I=1}(s_1)s_1^2 s_1^{(1)} - \right.$$

$$\frac{1}{12}[R_{PT}^{I=1}(s_1)s_1^2 s_1^{(1)}]^{(1)}\Bigg\} \tag{19}$$

We discard the terms with small coefficients $1/720$, $1/30240$ in formulae (17)-(19). Let us write $R_{PT}^{I=1}$ in the form

$$R_{PT}^{I=1}(s) = \frac{3}{2}(1 + r(s)) \tag{20}$$

Function $r(s)$ is calculated in [6]. Neglecting the terms associated with u- and d-quark masses we get

$$C_2 = \frac{1}{8\pi^2}\Bigg\{-s_1+s_0^{(1)}+\frac{1}{2}s_1^{(1)}-\frac{1}{12}s_1^{(2)}-\int_0^{s_1} r(s)ds+r(s_0)s_0^{(1)}+\frac{1}{2}r(s_1)s_1^{(1)}-\frac{1}{12}[r(s_1)s_1^{(1)}]^{(1)}\Bigg\} = 0 \tag{21}$$

$$C_4 = \frac{1}{8\pi^2}\Bigg\{\frac{1}{2}s_1^2 - s_0 s_0^{(1)} - \frac{1}{2}s_1 s_1^{(1)} + \frac{1}{12}(s_1 s_1^{(1)})^{(1)} + \int_0^{s_1} sr(s)ds - s_0 s_0^{(1)} r(s_0) -$$

$$-\frac{1}{2}s_1 s_1^{(1)} r(s_1) + \frac{1}{12}[s_1 s_1^{(1)} r(s_1)]^{(1)}\Bigg\} \tag{22}$$

$$C_6 = \frac{1}{8\pi^2}\Bigg\{-\frac{1}{3}s_1^3 + s_0^2 s_0^{(1)} + \frac{1}{2}s_1^2 s_1^{(1)} - \frac{1}{12}(s_1^2 s_1^{(1)})^{(1)} - \int_0^{s_1} s^2 r(s)ds +$$

$$+s_0^2 s_0^{(1)} r(s_0) + \frac{1}{2}s_1^2 s_1^{(1)} r(s_1) - \frac{1}{12}(s_1^2 s_1^{(1)} r(s_1))^{(1)}\Bigg\} \tag{23}$$

Using (21) to eliminate the unobservable quantity $s_1^{(2)}$ we reduce (22) and (23) to the form

$$\langle 0 \mid \frac{\alpha_s}{\pi}G^2 \mid 0\rangle = \frac{3}{\pi^2}\Bigg\{-\frac{1}{2}s_1^2 + (s_1 - s_0)s_0^{(1)} + \frac{1}{12}s_1^{(1)^2} - \int_0^{s_1}(s_1 - s)r(s)ds$$

$$+(s_1 - s_0)s_0^{(1)} r(s_0) + \frac{1}{12}s_1^{(1)^2} r(s_1)\Bigg\} \tag{24}$$

$$C_6 = \frac{1}{8\pi^2}\Bigg\{\frac{2}{3}s_1^3 - (s_1^2 - s_0^2)s_0^{(1)} - \frac{1}{6}s_1 s_1^{(1)^2} + \int_0^{s_1}(s_1^2 - s^2)r(s)ds -$$

$$-(s_1^2 - s_0^2)s_0^{(1)} r(s_0) - \frac{1}{6}s_1 s_1^{(1)^2} r(s_1)\Bigg\} \tag{25}$$

Let us calculate $s_0^{(1)}$ and $s_1^{(1)}$.
We obtain from (12) and (20)

$$s_k^{(1)} = \frac{6\pi\Gamma_k^{ee}M_k}{\alpha^2(1 + r(s_k))} \tag{26}$$

We know only Γ_0^{ee}, therefore

$$s_0^{(1)} = (1.554 \pm 0.024)\ GeV^2 \quad (\Lambda_3 = 1.565\ GeV, \quad r(s_0) = 0.204) \tag{27a}$$

$$s_0^{(1)} = (1.634 \pm 0.026)\ GeV^2 \quad (\Lambda_3 = 0.618\ GeV, \quad r(s_0) = 0.144) \qquad (27b)$$

The quantity $s_1^{(1)}$ is determined from trivial equations

$$s_2 = s_1 + s_1^{(1)} + \frac{1}{2}s_1^{(2)} + \frac{1}{6}s_1^{(3)} \qquad (28)$$

$$s_0 = s_1 - s_1^{(1)} + \frac{1}{2}s_1^{(2)} - \frac{1}{6}s_1^{(3)} \qquad (29)$$

From (28) and (29) we obtain

$$s_1^{(1)} = \frac{s_2 - s_0}{2} - \frac{1}{6}s_1^{(3)} \qquad (30)$$

and

$$s_1^{(2)} = s_0 + s_2 - 2s_1 = -0.81 \pm 0.16\ GeV^2 \qquad (31)$$

To estimate the last term in (30), we note that $|s_1^{(2)}|$ is smaller than $s_1^{(1)}$ (see 32)). We put $|s_1^{(3)}| = |s_1^{(2)}|$ and include the last term in (30) into the error in $s_1^{(1)}$ and obtain finally

$$s_1^{(1)} = \frac{s_2 - s_0}{2} = 1.148 \pm 0.139\ GeV^2 \qquad (32)$$

Using eqs.(24) and (25) and the values of $s_0^{(1)}$ and $s_1^{(1)}$ from (27a, 27b, 32), we find that the analysis of the ρ-meson family leads in the 3-loop approximation to the following results for gluon and four-quark condensates

$$\langle 0 \mid \frac{\alpha_s}{\pi}G^2 \mid 0 \rangle = (0.0744 \pm 0.0227)GeV^4, \quad (\Lambda_3 = 1.565\ GeV) \qquad (33a)$$

$$\langle 0 \mid \frac{\alpha_s}{\pi}G^2 \mid 0 \rangle = (0.112 \pm 0.021)GeV^4, \quad (\Lambda_3 = 0.618\ GeV) \qquad (33b)$$

$$C_6 = -0.0072 \pm 0.0041 GeV^6, \qquad (\Lambda_3 = 1.565\ GeV) \qquad (34a)$$

$$C_6 = -0.0115 \pm 0.0034 GeV^6, \qquad (\Lambda_3 = 0.618\ GeV) \qquad (34b)$$

The errors presented in (33a - 34b) were found from the formulae

$$\Delta \langle 0 \mid \frac{\alpha_s}{\pi}G^2 \mid 0 \rangle = \frac{3}{\pi^2}\left\{ \left[(s_1 - s_0)(1 + r(s_0))\Delta s_0^{(1)} \right]^2 + \right.$$

$$\left. + \left[\left(s_0^{(1)} - s_1 - \int_0^{s_1} r(s)ds + s_0^{(1)}r(s_0) \right)\Delta s_1 \right]^2 + \left[\frac{1}{6}s_1^{(1)}(1 + r(s_1))\Delta s_1^{(1)} \right]^2 \right\}^{1/2} \qquad (35)$$

$$\Delta C_6 = \frac{1}{8\pi^2}\left\{ \left[\left(2s_1^2 - 2s_1 s_0^{(1)} - \frac{1}{6}s_1^{(1)^2} + 2s_1 \int_0^{s_1} r(s)ds \right)\Delta s_1 \right]^2 + \right.$$

$$\left. + \left[(s_1^2 - s_0^2)(1 + r(s_0))\Delta s_0^{(1)} \right]^2 + \left[\frac{1}{3}s_1 s_1^{(1)}(1 + r(s_1))\Delta s_1^{(1)} \right]^2 \right\}^{1/2} \qquad (36)$$

The symbol Δ in equations in (35) and (36) denotes the error in the corresponding quality (for example, $\Delta s_1^{(1)} = 0.139 GeV^2$). The small error connected with the error in Λ_3 is taken into account at numerical calculations.

IV. MAGNITUDE OF GLUON CONDENSATE FROM THE ANALYSIS OF THE ω-MESON FAMILY

The polarization operator associated with the isoscalar current $j_\mu^{I=0}(x)$ of light quarks can be written as

$$i \int d^4x e^{iqx} \langle 0 \mid T\left\{ j_\mu^{I=0}(x) j_\nu^{I=0}(0) \right\} \mid 0 \rangle = (q_\mu q_\nu - q^2 g_{\mu\nu}) \Pi^{(w)}(Q^2) \tag{37}$$

where

$$j_\mu^{I=0}(x) = \frac{1}{6}\{\overline{u}(x)\gamma_\mu u(x) + \overline{d}(x)\gamma_\mu d(x)\} \tag{38}$$

With the exception of (12), all the equations presented above for the ρ-meson family remain in force. The equation that takes place in (12) for the ω-meson family is

$$\Gamma_k^{ee} = \frac{1}{9}\frac{\alpha^2}{3\pi}(1 + r(s_k))M_k^{(1)} \tag{39}$$

Three ω-mesons – the ω-meson with the mass $M_0 = 0.78257 \pm 0.00012 GeV$ and the electronic width $\Gamma_0^{ee} = 0.60 \pm 0.02 keV$, the ω'-meson with the mass $M_1 = 1.419 \pm 0.031 GeV$, and ω''-meson with the mass $M_2 = 1.649 \pm 0.024 GeV$ [30] – have been discovered thus far. Proceeding in the same way as for ρ-meson family and taking into account (39), we obtain

$$s_0^{(1)} = (1.244 \pm 0.041) \ GeV^2, \quad \Lambda_3 = 1.565 GeV \tag{40a}$$

$$s_0^{(1)} = (1.308 \pm 0.044) \ GeV^2, \quad \Lambda_3 = 0.618 GeV \tag{40b}$$

$$s_1^{(1)} = (1.053 \pm 0.123) \ GeV^2 \tag{40}$$

Using equations (24) and (35) and the values in (40a,40b,40) we obtain

$$\langle 0 \mid (\alpha_s/\pi)G^2 \mid 0 \rangle = (-0.076 \pm 0.033) \ GeV^4 \quad (\Lambda_3 = 1.565 \ GeV) \tag{41a}$$

$$\langle 0 \mid (\alpha_s/\pi)G^2 \mid 0 \rangle = (-0.043 \pm 0.031) \ GeV^4 \quad (\Lambda_3 = 0.618 \ GeV) \tag{41b}$$

$$C_6 = (0.0079 \pm 0.0052)GeV^6 \quad (\Lambda_3 = 1.565 \ GeV) \tag{42a}$$

$$C_6 = (0.0042 \pm 0.0045)GeV^6, \quad (\Lambda_3 = 0.618 \ GeV) \tag{42b}$$

The magnitude of the gluon condensate obtained from different processes must be equal. We found that the magnitude of the gluon condensate determined from the analysis of the ω-meson family is inconsistent with the magnitude of the gluon condensate from the analysis of the ρ-family. The way out of this situation is proposed in the next sections.

V. $\rho - \omega$-INTERFERENCE AND RESOLUTION
OF THE CONTRADICTION

Equations (37)-(39) are valid for isospin-zero vector mesons. However, there is a noticeable isospin-1 admixture in the real ω-meson. This is obvious, for example, from the fact that the width $(\Gamma(\omega \to 2\pi)$ is nonzero,

$$\Gamma(\omega \to 2\pi) = 0.143 \pm 0.024 MeV \quad [30]$$

Moreover, the ratio of the electronic widths of ρ and ω mesons is $(\Gamma_\rho^{ee}/\Gamma_\omega^{ee} = 11.42(1 \pm 0.037)$, instead of the expected value 9. Let us represent the ω-meson and ρ states as

$$|\omega\rangle = |\omega_0\rangle + \lambda |\rho_0\rangle \tag{43}$$

$$|\rho\rangle = |\rho_0\rangle - \lambda |\omega_0\rangle$$

where $|\omega_0\rangle$ corresponds to the I= 0 state and $|\rho_0\rangle$ corresponds to the I= 1 state. The mixing parameter λ can be found in two ways

1) It follows from the formula $\langle e^+e^- |\rho_0\rangle = 3\langle e^+e^- |\omega_0\rangle$ and the formulae (43) that:

$$\langle e^+e^- |\omega\rangle = (1+3\lambda)\langle e^+e^- |\omega_0\rangle \tag{44}$$

$$\langle e^+e^- |\rho\rangle = (3-\lambda)\langle e^+e^- |\omega_0\rangle \tag{45}$$

Division of eq.(45) by eq.(44) gives

$$\frac{3-\lambda}{1+3\lambda} = \frac{\langle e^+e^- |\omega\rangle}{\langle e^+e^- |\rho\rangle} = \sqrt{\frac{\Gamma_\rho^{ee}}{\Gamma_\omega^{ee}}} = 3.379 \pm 0.063 \tag{46}$$

It follows from Eq.(46), that

$$\lambda = -0.034 \pm 0.005 \tag{47}$$

It follows Eqs.(43), that

$$\langle \pi^+\pi^- |\omega\rangle = \lambda\langle \pi^+\pi^- |\rho_0\rangle$$

$$\langle \pi^+\pi^- |\rho\rangle = \langle \pi^+\pi^- |\rho_0\rangle \tag{48}$$

The value λ following from Eq.(48) is

$$\lambda = \frac{\langle \pi^+\pi^- |\omega\rangle}{\langle \pi^+\pi^- |\rho\rangle} = -\sqrt{\frac{\Gamma(\omega \to 2\pi)}{\Gamma(\rho \to 2\pi)}} = -0.031 \pm 0.0026 \tag{49}$$

The values $\Gamma(\omega \to 2\pi)$ and $\Gamma(\rho \to 2\pi)$ are taken Particle Data [30]. In the narrow – resonance approximation the parameter λ must be real. The values of λ from Eqs.(47) and (49) are in good agreement. Finally we obtain the averaged mixing parameter $\overline{\lambda}$

$$\overline{\lambda} = 0.0316 \pm 0.0023 \tag{50}$$

It follows from Eq.(44) that

$$\Gamma_{\omega_0}^{ee} = \frac{\Gamma^{ee}}{1+6\lambda} = (0.741 \pm 0.027) \, keV \tag{51}$$

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Instead of the values $s_0^{(1)}$ from Eqs.(40a),(40b) we get the new values

$$s_0^{(1)} = (1.535 \pm 0.057)\ GeV^2, \quad (\Lambda_3 = 1.565\ GeV) \qquad (52a)$$

$$s_0^{(1)} = (1.596 \pm 0.026)\ GeV^2, \quad (\Lambda_3 = 0.618\ GeV) \qquad (52b)$$

corresponding to the contribution of the isospin $I = 0$ in the ω-meson.

The magnitude of gluon condensate following from the analysis of the ω-family is

$$\langle 0 \mid \frac{\alpha_s}{\pi} G^2 \mid 0 \rangle = (0.073 \pm 0.034)\ GeV^4, \quad (\Lambda_3 = 1.565\ GeV) \qquad (53a)$$

$$\langle 0 \mid \frac{\alpha_s}{\pi} G^2 \mid 0 \rangle = (0.108 \pm 0.033)\ GeV^4, \quad (\Lambda_3 = 0.618\ GeV) \qquad (53b)$$

This GC magnitude is consistent with the value which follows from the analysis of the ρ-meson family.

For C_6 we have

$$C_6 = (-0.0084 \pm 0.0050)\ GeV^6, \quad (\Lambda_3 = 1.565\ GeV) \qquad (54a)$$

$$C_6 = (-0.0123 \pm 0.0044)\ GeV^6, \quad (\Lambda_3 = 0.618\ GeV) \qquad (55b)$$

The results of the calculations of gluon and 4-quarks condensates in 0–3 loops approximation from the analysis of ω-meson family are presented in the Tables I,II

TABLE I.   The results of the calculations of the gluon and 4-quarks condensates in the 0–3 loops approximation for Λ_3^{new} from analysis of ω-meson family taking into account the $\rho - \omega$ interference.

	Λ_3^{new}/GeV	$\frac{\langle 0\mid(\alpha_s/\pi)G^2\mid\rangle}{GeV^4}$	C_6/GeV^6	$r(m_\omega^2)$	$r(m_{\omega'}'^2)$
0 loops		0.198±0.100	-0.0218±0.0034	0	0
1 loop	0.618± 0.059	0.070±0.034	-0.0082±0.0050	0.201±0.008	0.153±0.007
2 loops	1.192±0.136	0.072± 0.034	-0.0083± 0.005	0.203±0.008	0.161±0.008
3 loops	1.565±0.193	0.073±0.034	-0.0084±0.0050	0.202± 0.008	0.164±0.008

TABLE II.   The results of the calculations of the gluon and 4-quarks condensates in the 0–3 loops approximation for Λ_3^{conv} from the analysis of ω-meson family taking into account the $\rho - \omega$ interference.

	Λ_3^{conv}/GeV	$\frac{\langle 0\mid(\alpha_s/\pi)G^2\mid\rangle}{GeV^4}$	C_6/GeV^6	$r(m_\omega^2)$	$r(m_{\omega'}'^2)$
0 loops		0.198±0.100	-0.0218±0.0034	0	0
1 loops	0.370±0.019	0.097±0.033	-0.0111±0.0046	0.159±0.004	0.122±0.003
2 loops	0.539±0.025	0.105±0.033	-0.0119±0.0045	0.148±0.003	0.114±0.002
3 loops	0.618±0.29	0.108±0.033	-0.0123± 0.044	0.143±0.003	0.111±0.002

VI. The magnitude of gluon and four-quark condensates
from ρ-meson family with the account of $\rho - \omega$-interference

Because of $\lambda \neq 0$ there is a small admixture of isospin $I = 0$ into $\rho(770)$ meson. The electronic width of $\rho(770)$-meson due to isospin $I = 1$ is equal to

$$\Gamma_{\rho_0}^{ee} = \frac{\Gamma_\rho^{ee}}{(1 - \frac{\lambda}{3})^2} = 6.71 \pm 0.11 keV \tag{56}$$

The corrected value $s_0^{(1)}$ is equal to

$$s_0^{(1)} = (1.521 \pm 0.025)GeV^2, \quad (\Lambda_3 = 1.565 GeV) \tag{57a}$$

$$s_0^{(1)} = (1.596 \pm 0.026)GeV^2, \quad (\Lambda_3 = 0.618 GeV) \tag{57b}$$

The corresponding GC magnitude is

$$\langle 0|\frac{\alpha_s}{\pi}G^2|0\rangle = (0.056 \pm 0.023)GeV^4 \quad (\Lambda_3 = 0.565 GeV) \tag{58a}$$

$$\langle 0|\frac{\alpha_s}{\pi}G^2|0\rangle = (0.096 \pm 0.021)GeV^4, \quad (\Lambda_3 = 0.618 GeV) \tag{58b}$$

For C_6 we have

$$C_6 = (-0.0051 \pm 0.0045)GeV^6, \quad (\Lambda_3 = 1.565 GeV) \tag{59a}$$

$$C_6 = (-0.0097 \pm 0.0039)GeV^6, \quad (\Lambda_3 = 0.618 GeV) \tag{59b}$$

The results of the calculations of gluon and 4-quarks condensates in 0–3 loops approximation from analysis of the ρ-meson family are presented in the Tables III, IV.

TABLE III.   The results of the calculations of the gluon and 4-quarks condensates in the 0–3 loops approximation for Λ_3^{new} from analysis of ρ-meson family taking into account the $\rho - \omega$ interference.

	Λ_3^{new}/GeV	$\frac{\langle 0|(\alpha_s/\pi)G^2|\rangle}{GeV^4}$	C_6/GeV^6	$r(m_\rho^2)$	$r(m_{\rho'}^2)$
0 loops		0.197±0.059	-0.0211±0.0025	0	0
1 loop	0.618±0.059	0.049±0.023	-0.0044±0.0045	0.202± 0.008	0.151±0.007
2 loops	1.192±0.136	0.055±0.023	-0.0050±0.0045	0.204±0.008	0.159±0.008
3 loops	1.565±0.193	0.056±0.023	-0.0051±0.0045	0.203±0.008	0.162±0.008

TABLE IV.   The results of the calculations of the gluon and 4-quarks condensates in the 0–3 loops approximation for Λ_3^{conv} from analysis of ρ-meson family taking into account the $\rho - \omega$ interference.

	Λ_3^{new}/GeV	$\frac{\langle 0|(\alpha_s/\pi)G^2|\rangle}{GeV^4}$	C_6/GeV^6	$r(m_\rho^2)$	$r(m_{\rho'}^2)$
0 loops		0.197±0.059	-0.0211±0.0025	0	0
1 loop	0.370±0.019	0.070±0.034	-0.0082±0.0050	0.159± 0.006	0.151±0.007
2 loops	1.539±0.025	0.093±0.022	-0.0094±0.0039	0.149±0.003	0.113±0.002
3 loops	1.618±0.029	0.096±0.021	-0.097±0.039	0.144±0.003	0.109±0.002

Since $\rho(770)$ has a small admixture of isospin $I = 0$, the decay $\rho \to \pi^+\pi^-\pi^0$ exists with the width

$$\Gamma_{\rho\to\pi^+\pi^-\pi^0} = \lambda^2 \Gamma_{\omega\to\pi^+\pi^-\pi_0} = (7.23 \pm 1.20) keV \tag{60}$$

The value

$$\Gamma_{\rho\to\pi^+\pi^-\pi^0}/\Gamma_{tot} = (4.8 \pm 0.8) \cdot 10^{-5} \tag{61}$$

is smaller than the experimental restriction [30]

$$(\Gamma_{\rho\to\pi^+\pi^-\pi^0}/\Gamma_{tot})_{Exp} < 1.2 \cdot 10^{-4} \tag{62}$$

VII.   CALCULATION OF THE ELECTRONIC WIDTH $\rho(1450)$.

From eq.(26) we have

$$\Gamma_1^{ee} = \frac{\alpha^2}{6\pi} \frac{(1 + r(s_1))}{M_1} s_1^{(1)} \tag{63}$$

and using $s_1^{(1)}$ we get from (32)

$$\Gamma_1^{ee} = (2.57 \pm 0.31)\ keV, \quad (\Lambda_3 = 1.565\ GeV) \tag{64a}$$

$$\Gamma_1^{ee} = (2.46 \pm 0.30)\ keV, \quad (\Lambda_3 = 0.618\ GeV) \tag{64b}$$

The results presented in (54a,54b) are consistent with the value $\Gamma_1^{ee} = (2.5 \pm 0.9) keV$ obtained in [31].

VIII.   CALCULATION OF THE ELECTRONIC WIDTH $\omega(1420)$

We have from eq.(39)

$$\Gamma_1^{ee} = \frac{\alpha^2}{54\pi} \frac{(1 + r(s_1))}{M_1} s_1^{(1)} \tag{65}$$

and using $s_1^{(1)}$ from (40) we obtain

$$\Gamma_1^{ee} = (0.27 \pm 0.03)\ keV, \quad (\Lambda_3 = 1.565\ GeV) \tag{66a}$$

$$\Gamma_1^{ee} = (0.26 \pm 0.03)\ keV, \quad (\Lambda_3 = 0.618\ GeV) \tag{66b}$$

Within the errors, the electronic width of the $\omega(1420)$-meson is one-ninth of the electronic width of the $\rho(1450)$ and is inconsistent with the value $\Gamma_1^{ee} = 0.15 \pm 0.4)keV$ [31].

IX. CONCLUSION

In summary, we present the results of the averaging over the ρ and ω-families of the gluon and 4-quark condensates in the 0-3 loop approximation in Table V,VI.

TABLE V. The averaged results of the calculation of the gluon and 4-quarks condensates in 0–3 loops approximation for Λ_3^{new} from analysis of ρ and ω families taking into account the $\rho - \omega$ interference.

	Λ_3^{new}/GeV	$\langle 0 \mid (\alpha_s/\pi)G^2 \mid \rangle /GeV^4$	C_6/GeV^6
0 loops		0.198±0.100	-0.0218±0.0034
1 loop	0.618±0.059	0.056±0.019	-0.0061±0.0033
2 loops	1.192±0.136	0.061±0.019	-0.0065±0.0034
3 loops	1.565±0.193	0.062±0.019	-0.0066±0.0034

TABLE VI. The averaged results of the calculation of the gluon and 4-quarks condensates in 0–3 loops approximation for Λ_3^{new} from analysis of ρ and ω families taking into account the $\rho - \omega$ interference.

	Λ_3^{new}/GeV	$\langle 0 \mid (\alpha_s/\pi)G^2 \mid \rangle /GeV^4$	C_6/GeV^6
0 loops		0.198±0.100	-0.0218±0.0034
1 loop	0.370±0.019	0.088±0.018	-0.0096±0.0030
2 loops	0.539±0.025	0.096±0.018	-0.0105±0.0029
3 loops	1.618±0.029	0.100±0.018	-0.0108±0.0029

It is seen from Tables I-VI that the expansion of the gluon and 4-quark condensates in terms of α_s is very good. The good convergence in α_s is due to the improved perturbative theory [52,50,6] used in the paper.

The magnitude of the gluon condensate from the analysis of families of J/Ψ and Υ mesons was obtained in paper [3].

$$0.04 \leq \langle (\alpha_s/\pi)G^2 \rangle \leq 0.105 \ GeV \qquad (67)$$

The first-order terms in α_s and Coulomb terms of all orders in α_s/v_k are taken into account when obtaining (67). The result (67) is in a lightly better agreement with Λ_3^{new} than with Λ_3^{conv}).

The conventional magnitude of gluon condensate is the value obtained by Shifman, Vainstein and Zakharov in their basic paper [11] from analysis of the J/Ψ family

$$\langle (\alpha_s/\pi)G^2 \rangle = 0.012 \ GeV^4 \qquad (68)$$

But as was shown in ref.[3] the model used by Shifman et al. in [11] to describe the experimental function $R_c(s)$ in the form of the sum of δ-functions (related to

the observed resonances from the J/ψ family) and a plateu contradicts the Wilson operator expansion in terms which are due to the gluon condensate. Later, there were many attempts to determine the gluon condensate by considering various processes within various approaches [32-51]. But in these works either R_c contradicts the Wilson operator expansion or Λ_3 is very small, $\Lambda_3 \sim 100 MeV$.

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